

**Protecting Equity Investments:
Options, Inverse ETFs, Hedge Funds, and AORDA Portfolios**

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Abstract

This paper discusses hedging strategies to protect long equity positions in different market conditions. The desirable property of a hedging strategy is to deliver non-negative returns (or positive returns) in rising markets and hedge, i.e., deliver profits covering losses of long positions, in down markets.

We show that buying put options provides hedging protection in declining markets, but it is an expensive strategy delivering deeply negative returns in the long run (the performance of such a strategy is illustrated with the Short PutWrite Index). Inverse Exchange Traded Funds (ETFs), similar to put options, have good hedging capabilities in falling markets but we show that these funds are also quite expensive in the long run due to compounding and liquidity effects (this is shown with the performance of SH which is an inverse ETF providing returns opposite to the S&P500).

We show that market volatility is the key factor for building profitable long term hedging strategies. Short volatility strategies benefit from low volatility levels, but during periods of high volatility may be completely wiped out. Long volatility strategies give high returns during market drawdowns when volatility is high but may have inferior performance in bull markets. We show, with an example, that a fund selling naked deep out-of-the money options is a short volatility strategy with spectacular returns in up and sideways markets while in down markets this strategy demonstrates catastrophic performance.

The majority of hedge fund styles are shorting volatility. However, research shows that long volatility strategies, such as hedge funds with a positive loading factor on market volatility, have good hedging capabilities. AORDA® Portfolio 2 is a long volatility strategy providing a statistical hedge for market drawdowns. This portfolio is designed to perform especially well during bear markets while delivering, on average, positive returns in bull markets, in contrast with long put options strategies and inverse ETFs. AORDA Portfolios are based on formal Conditional Value-at-Risk (CVaR) risk management optimization technology. CVaR technology is designed to constrain tail loss risk with the objective of growing asset value. AORDA Portfolio 3, which is a mix of Portfolio 2 and the S&P500, performs well in both up and down markets.

1. Introduction: Buy-and-Hold Investments and the Need for Hedging Strategies

Buy-and-hold is the most popular investment strategy: an investor buys securities (e.g., stocks) and holds them for several years. The basic assumption is that in the long term, the economy keeps growing, which results in an increase in equity prices and dividends. Short term fluctuations due to the business cycle are possible, but

in the long run the market will rise. The advocates of the buy-and-hold investment strategy also say that with its small number of trades, it saves on trading commissions and taxes. Buy-and-hold strategy finds a theoretical basis in the Efficient Market Hypothesis (EMH), according to which stock prices always include all available information and are priced correctly. As a result, short term stock price movements are completely random and it is not possible to predict them. If EMH holds, then active investment strategies do not add value and the best investment strategy is to buy a stock for the long run.

Fig. 1 shows a plot of about 30 years of the S&P500 index, used to represent the stock market as a whole, that experienced a long term growth. It was interrupted by moderate market drops which are quickly recovered, but also by large drops that took several years to recover.



Fig. 1: S&P500 1981 -02/2011. Source: Google Finance.

There is clearly a long term uptrend for the S&P500 from 1981 to 2000. The index reached a peak in April 2000 at 1516.35 points, followed by a downtrend, touching a bottom in September, 2002 at 827.37 points. It took about 5 years to recover from this drawdown. In October, 2007 the S&P500 peaked at 1557.59 points and 17 months later, the index dropped to 683.38 points in March, 2009. As of February 2011, the S&P500 has still not reached the two previous tops. It seems that from 1981 to 1998 the market was in a long term uptrend, followed by a period of large swings from 1998 to the present.

Taking into account management fees and inflation, the recent 12 years of market stagnation resulted in LARGE LOSSES for ordinary investors. Let us assume that an investor has used an advisor allocating his money to several stock mutual funds. Let us also assume 2% management fees per year (combined fees of the advisor and mutual funds). If we assume that inflation was about 3% per year, this resulted in the total loss of about 5% per year in constant (uninflated) dollars. With these assumptions, the total cumulative loss of purchasing power in constant dollars over the recent 12 years equals about 46% (indeed, $1-0.95^{12}=0.46$).

The main problem with the buy-and-hold strategy is the total lack of risk control that can result in huge losses. Market volatility is higher during drawdowns. Prices usually decrease quicker than they increase, and thus the absence of any risk management technique exposes a passive unmanaged portfolio, to large fluctuations that in few days can completely wipe out positive returns that took years to achieve. A buy-and-hold strategy may be extremely risky in declining markets.

Stock market performance is a leading indicator: the stock market usually begins to decline before the economy declines and it improves before the economy begins to pull out of a recession. During economic downturns, decline in stock market returns are accompanied by an increase in unemployment rate, a decrease

in real estate values, in salaries, and in the value of retirement accounts. Thus, during an economic downturn even if an investor has no positions in the stock market, his overall wealth is nonetheless decreasing. That is, his wealth has a positive correlation with market returns. Although authors of this paper in general agree with the EMH, they believe that many improvements can be done to reduce the dangerous dependence of wealth upon stock market returns.

The natural question is then how to hedge the value of long equity investments, direct and indirect, in a declining market. Further on we consider the S&P500 as the most representative equity market index for the purposes of this paper.

2. Hedging with Put Options and Portfolio Insurance

The strategy of buying protective out-of-the money (OTM) put options is an effective insurance against market downturns, although it is a very expensive protection over the long term, as is any other insurance policy. Returns and losses of equity markets are not symmetric: a price fall is bad news for the shareholder while a price rise is good news. Traders usually increase the price of OTM put options during a period of high volatility and they can do that because there is a high demand for the insurance offered by these puts from risk-averse investors. Prices of OTM puts became expensive (in terms of implied volatility) after the 1987 Black Monday, when the largest one-day percentage market decline in history was recorded and that took several years to recover.

Portfolio insurance is a hedging strategy against declining markets, under which optimal stock-to-cash ratios are computed for various market prices. The investor decreases the weight on stocks during falling markets and increases the weight of stocks during rising markets. This is similar to buying a put option on these portfolios, since it allows an investor to preserve upside gains while limiting downside risk. A popular variant of the portfolio insurance strategy is known as Constant Proportion Portfolio Insurance (CPPI), in which the asset allocation must guarantee the minimum dollar value (floor) of the portfolio set by the investor. The CPPI uses risky assets (stocks, mutual funds...) and a riskless asset (cash, government bonds...). The percentage allocated to each depends on the so called cushion value, which is a multiplier of the difference between current portfolio value and the floor. The major drawback of the portfolio insurance strategy is that it forces the investor to buy during rising markets and to sell during declining markets, and when several institutional investors do it simultaneously, this artificially inflates the market movement in place with a snowball effect, and also creates serious liquidity problems. Portfolio insurance strategies are deemed to have a major role in the market crash of October 1987.

To evaluate the cost of hedging with put options (and with portfolio insurance which is another kind of put option), we consider the Chicago Board Options Exchange (CBOE) S&P 500 PutWrite Index. The PutWrite Index records profits of a portfolio selling at-the-money put options on a monthly basis (we provide a detailed description of PutWrite Index at the end of this section). The reader will observe that profits from this Index are higher than profits from the S&P500 index (at least in the recent 5 years). Suppose that for protecting purposes you buy at-the-money put options on a monthly basis. This means that you opened a SHORT position in the PutWrite Index. Payoffs of the Short PutWrite Index have an opposite sign when compared to the payoffs from the PutWrite Index. We plot the returns of the Short PutWrite Index against S&P500 (ticker: SPX) returns in Fig 2.

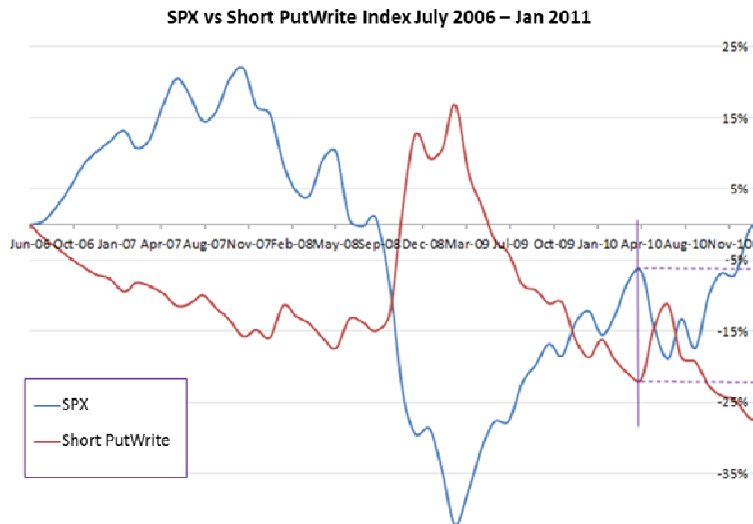


Fig.2: S&P500 (SPX) vs Short PutWrite Index. Data source for SPX and PutWrite Index : www.cboe.com

The reader should note the purple vertical line in Fig.2. At that point, the S&P500 was losing about 6% while the Short PutWrite Index, that is the hedging index, was losing more than 20%. At the right end of the graph, corresponding to January 2011, the SPX is slightly positive while the Short Putwrite Index is still losing more than 25%. Short PutWrite Index evaluates only a LOW BOUND of the hedging costs with options because the index does not take into account market frictions (transaction costs, and bid-ask spreads, and taxes). The graph shows that the long-term insurance of the Short PutWrite Index is expensive and completely “eats away” profits generated by the S&P500. We can state that long put option strategies provide protection against market drawdowns but these strategies have significant negative return over the long term and protection “consumes” positive returns generated by long positions.

Since June 2007 the CBOE has published daily values of the CBOE S&P 500 PutWrite Index (ticker symbol PUT)¹. PUT is a benchmark index that measures the performance of a hypothetical portfolio that sells S&P 500 Index (SPX) put options against collateralized cash reserves held in a money market account. The PUT strategy is designed to sell a sequence of one-month, at-the-money, S&P 500 Index puts and invest cash at one and three-month Treasury Bill rates. The number of puts sold varies from month to month, but is limited so that the amount held in Treasury Bills can finance the maximum possible loss from final settlement of the SPX puts. That is, the number of puts is set to collateralize the exposure to S&P 500 downturns. This design provides high leverage, and it can also capture the “rich” premium of S&P 500 put options documented in several academic studies². These studies have found that short option strategies, and especially short put strategies, appear to generate high risk-adjusted returns. Reasons cited for the excess returns are the negative risk-premium gathered by volatility, and, in the case of puts, the high demand for portfolio protection.

Since 1986, the PUT Index has earned higher returns than the S&P 500, with lower volatility. The PUT Index tends to outperform in quiet and falling markets, and underperform when stock prices rise sharply. Long put options insure against the decline of a stock market index below the stated strike price. If the stock market does not fall during the life of the put option, the put seller keeps the entire premium. If the stock market declines below the strike price at expiration, the put seller pays the put buyer the difference between the

¹ See www.cboe.com/micro/put/, www.cboe.com/micro/put/PutWrite.pdf and references therein.

² See www.cboe.com/micro/put/PutWrite.pdf

stock index value and the strike price. When stock markets rise, the return is limited to the amount of premium income. When stock markets decline, the seller takes the downside risk of the equity market, but outperforms an investment in the stock market index by the amount of the premium earned from the sale of the put. Fig.3 compares returns of PUT and S&P500 indices since 2005.

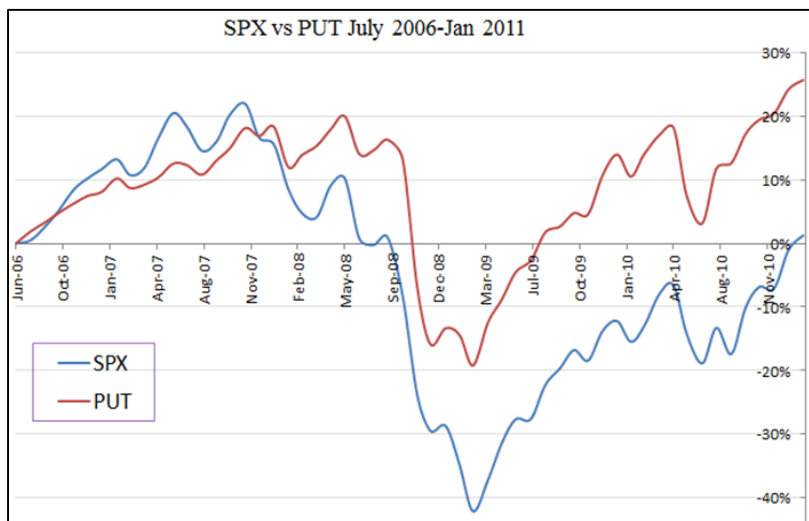


Fig. 3: CBOE S&P500 PutWrite Index vs. S&P500 Index. Data source: www.cboe.com

3. Hedging with Inverse ETFs

Nowadays, various Exchange Traded Funds (ETFs) allow investors to trade broad market indices, taking long, short (inverse) or leveraged positions. For example, to open a position in the S&P500 you can invest in the SPY to take a long position, in the SSO to take a long position with leverage two, in the SH for short position and in the SDS for short position with leverage two. The unleveraged ETFs that mimic the S&P500, seek a return of 100% (or -100%) of the return of the S&P500 on any single day. However, the compounding of daily returns of these ETFs over long periods differs significantly from the target returns. The following example illustrates the compounding effect. Let x be the 1-day return on the S&P500, and suppose that one day the index goes up 10% and the next day goes down by the same amount. The compounded return over 2 days would be $(1+x)*(1-x)=1-x^2=0.99$. This effect impacts dramatically the returns of long and short ETFs. Leverage further amplifies the compounding effect, and so do the ETFs management fees.

There are liquidity problems for inverse ETFs during crises times when hedging protection is especially needed. Fig.4 shows the performance of the S&P500 versus SH since October 2006. At the beginning of 2010 (see the purple vertical line in Fig. 4), the S&P500 was losing about 10% since the beginning of the considered period, while the SH was losing around 25% over the same time. At the right end of the graph, representing February 2011, the S&P500 was slightly below 10% while the SH was at -40%. This graph shows that the SH is not a good long term hedge against S&P500 drawdowns, although this ETF provides excellent hedging capabilities on a short term basis. In general we can state that inverse ETFs provide protection against market drawdowns but they have negative long term returns.



Fig.4: S&P500 vs SH since July 2006. Source: Yahoo! Finance

4. Hedge Funds: Positive and Negative Volatility Exposure

Many hedge funds claim that their returns have a low correlation with the market returns. For instance, so called Market Neutral hedge funds are supposed to generate a positive return both in up and down markets. Some academic studies show (Bondarenko (2004)) that for most categories of hedge funds a significant fraction of returns can be explained by a negative loading on a volatility factor. i.e., the majority of hedge funds SHORT volatility. Lo (2001, 2010) describes a hypothetical hedge fund, "Capital Decimation Partners", which shorts out-of-the-money S&P500 put options on each monthly expiration date with strikes approximately 7% out of the money. This hedge fund realizes exceptional performance in sideways and up markets, when market volatility is low. However, this fund has large losses when the market goes down sharply and when volatility is high.

Tab. 1 reports performance statistics for "Capital Decimation Partners". Over the eight year time period, the fund has a higher monthly mean return, a higher Sharpe Ratio, and a lower number of negative months than the S&P500. However, during August and September 1998, and October 1999, the S&P500 experienced quite large losses. The considered hedge fund amplified those losses. This example further shows that the strategy of shorting put options on the S&P500 results in positive returns most of the time, but it has extreme losses, when large market drops occur. The considered fund SHORTS volatility and it is exposed to market tail risk.

Capital Decimation Partners, L.P.

Performance Summary, January 1992 to December 1999

Statistic	S&P 500	CDP
Monthly Mean	1.4%	3.7%
Monthly Std. Dev.	3.6%	5.8%
Min Month	-8.9%	-18.3%
Max Month	14.0%	27.0%
Annual Sharpe Ratio	0.98	1.94
# Negative Months	36/96	6/96
Correlation with S&P 500	100.0%	59.9%
Total Return	367.1%	2721.3%

Tab. 1: performance summary for shorting OTM put strategy. Source: Lo (2001, 2010)

Capital Decimation Partners, L.P.
Monthly Performance History

Month	1992		1993		1994		1995		1996		1997		1998		1999	
	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP	SPX	CDP
Jan	8.2	8.1	-1.2	1.8	1.8	2.3	1.3	3.7	-0.7	1.0	3.6	4.4	1.6	15.3	5.5	10.1
Feb	-1.8	9.3	-0.4	1.0	-1.5	0.7	3.9	0.7	5.9	1.2	3.3	6.0	7.6	11.7	-0.3	16.6
Mar	0.0	4.9	3.7	3.6	0.7	2.2	2.7	1.9	-1.0	0.6	-2.2	3.0	6.3	6.7	4.8	10.0
Apr	1.2	3.2	-0.3	1.6	-5.3	-0.1	2.6	2.4	0.6	3.0	-2.3	2.8	2.1	3.5	1.5	7.2
May	-1.4	1.3	-0.7	1.3	2.0	5.5	2.1	1.6	3.7	4.0	8.3	5.7	-1.2	5.8	0.9	7.2
Jun	-1.6	0.6	-0.5	1.7	0.8	1.5	5.0	1.8	-0.3	2.0	8.3	4.9	-0.7	3.9	0.9	8.6
Jul	3.0	1.9	0.5	1.9	-0.9	0.4	1.5	1.6	-4.2	0.3	1.8	5.5	7.8	7.5	5.7	6.1
Aug	-0.2	1.7	2.3	1.4	2.1	2.9	1.0	1.2	4.1	3.2	-1.6	2.6	-8.9	-18.3	-5.8	-3.1
Sep	1.9	2.0	0.6	0.8	1.6	0.8	4.3	1.3	3.3	3.4	5.5	11.5	-5.7	-16.9	-0.1	8.3
Oct	-2.6	-2.8	2.3	3.0	-1.3	0.9	0.3	1.1	3.5	2.2	-0.7	5.6	3.6	27.0	-6.6	-10.7
Nov	3.6	8.5	-1.5	0.6	-0.7	2.7	2.6	1.4	3.8	3.0	2.0	4.6	10.1	22.8	14.0	14.5
Dec	3.4	1.2	0.8	2.9	-0.6	10.0	2.7	1.5	1.5	2.0	-1.7	6.7	1.3	4.3	-0.1	2.4
Year	14.0	46.9	5.7	23.7	-1.6	33.6	34.3	22.1	21.5	28.9	26.4	84.8	24.5	87.3	20.6	105.7

Tab. 2: monthly performance for shorting OTM put strategy. Source: Lo (2001, 2010)

In 1993 the CBOE introduced a volatility index, the VIX, based on real-time prices of options on the S&P500 Index, listed on the CBOE. It was designed to reflect investors' view of future (30-day) expected stock market volatility. Fig. 5 shows the S&P500 and the VIX in the last 5 years³.



Fig. 5: Volatility index VIX (red) vs. S&P500. Source: Yahoo!Finance

Volatility, as measured by VIX, is very volatile. As shown in Tab.3 below, the volatility of the VIX Index was higher than the volatility of VX Near-Term futures, S&P 500 Index (SPX), the Nasdaq-100 Index (NDX) and the Russell 2000 Index (RUT), and several stocks, including Google and Apple.

³ <http://www.cboe.com/micro/VIX/vixintro.aspx>

Name	12/31/08 Price	2008 Volatility	Name	12/31/09 Price	2009 Volatility
VIX	40.00	127.3%	VIX	21.68	88.9%
VX Near-Term Futures	41.94	88.9%	VX Near-Term Futures	22.95	69.2%
SPX	903.25	41.0%	SPX	1,115.10	27.3%
NDX	1,211.65	42.3%	NDX	1,860.31	26.5%
RUT	499.45	46.4%	RUT	625.39	36.2%
GOOG	307.65	55.2%	GOOG	619.98	30.1%
AAPL	85.35	58.2%	AAPL	210.73	33.7%

Tab. 3: Volatilities of various securities in 2008 and 2009. Source: www.cboe.com

There is a negative correlation between VIX and S&P500: when VIX rises, stock prices fall, and as the VIX falls, stock prices rise. Fig.6 shows negative correlation of VIX and S&P500 returns.

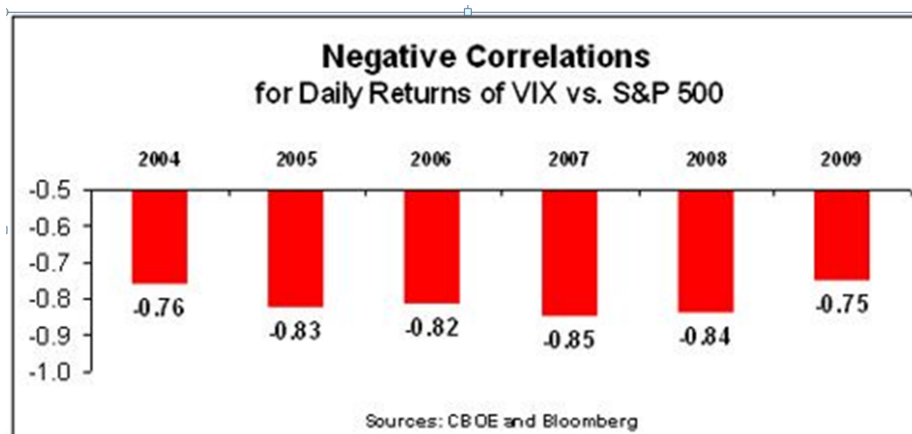


Fig.6: Negative daily correlation for volatility index VIX and S&P500. Source: www.cboe.com

Thus, returns of the S&P500 have a volatility that has by itself a high volatility. In addition, this volatility of volatility is higher during markets drawdowns. Volatility risk is the risk of loss due to changes in volatility. A trading position that benefits from an increase in volatility is long volatility. For example, since a long option is a long volatility position, option prices increase when volatility increases. Conversely, a short option sells the volatility risk, so a short option benefits from low levels of volatility.

It is well known that the volatility of equity returns is stochastic; however it is less understood whether volatility risk is compensated and whether this compensation is higher or lower than the risk-free rate. Bakshi and Kapadia (2003), and Bondarenko (2004) investigate whether the volatility risk premium is priced in the equity index options market and they find evidence that option prices reflect a negative market volatility risk premium. It has been argued in several studies that equity hedge funds have nonlinear, option-like payoffs with respect to the market return. In particular, Agarwal and Naik (2004) have found that many hedge fund categories exhibit returns similar to those from selling put options, and thus have a negative exposure to volatility risk. Strategies that sell volatility make small profits in calm markets but take big losses during market disruptions. Bondarenko (2004) shows that most hedge funds routinely sell volatility risk and that the exposure to volatility risk accounts for a considerable portion of hedge funds' average returns. After correcting for the short volatility risk exposure, the performance of most hedge funds categories becomes much less impressive, with positive alphas often becoming negative or statistically insignificant. Hedge funds having long volatility exposure provide good hedging protection for investors because they have high returns when the market goes down and when volatility is high. For instance, dedicated short bias (DSB) hedge funds, for which short selling is the main source of return and, as such, have positive performance when the markets fall, exhibited extremely strong results during the 2007 and 2008 market downturn. Connolly and Hutchinson (2010) show

that DSB hedge funds are a significant source of diversification for investors and produce statistically significant levels of alpha.

5. AORDA Portfolios

The main idea behind the American Optimal Advisors (AOA) investment approach (see, www.aorda.com/aoa) is to provide a statistical hedge for market drawdowns. AOA has created highly disciplined, active investment strategies having negative or low correlation with the market; these strategies perform especially well during bear markets while delivering positive returns, on average, in bull markets. AOA is using the AORDA® trademark for naming portfolios. AORDA portfolios are based on a completely automatic, quantitative day-trading strategy that combines both long and short positions in the S&P 500 and the NASDAQ 100. Using various instruments tracking these indices (e.g., mini-futures ES and NQ, or SPY and QQQQ, or index tracking funds in the RYDEX family of funds). AOA manages three AORDA portfolios in which a certain percentage of the total capital is invested in an active trading strategy, while the remaining capital is in cash or a long position in an S&P500 index.

AORDA Portfolios 1 and 2 are negatively correlated with the market; their goal is to achieve high positive returns (exceeding market loss) in bear markets and still achieve positive returns in bull markets. Portfolio 2 has two times higher return and risk than Portfolio 1. Portfolios 1 and 2 are designed to hedge against market loss and are not designed to always outperform the market. Portfolio 3 is a mix of Portfolio 2 and the market and is slightly positively correlated with the market. Portfolio 3 is intended to have positive returns in all market conditions.

Fig.7 shows all negative quarterly returns of the S&P500 versus AORDA Portfolio 2 for the period January 2005 - December 2010. In all quarters when market return was negative (i.e., when market went down) Portfolio 2 had a positive return.

Fig.8 shows all positive quarterly returns of the S&P500 versus AORDA Portfolio 2 for the period January 2005 - December 2010. When the market is up, AORDA portfolio 2 had slightly positive return on average. However, Portfolio 2 has tendency to lose when the market has especially high returns.

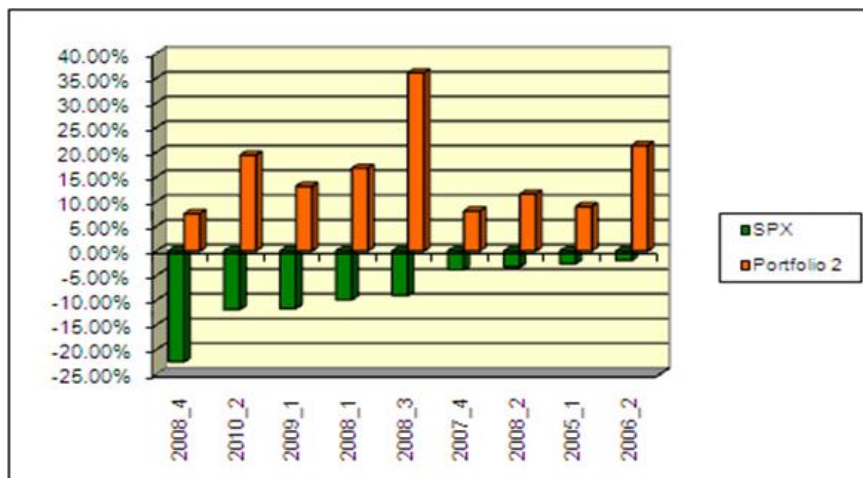


Fig.7: All negative quarterly returns of S&P500 and AOA Portfolio2.
 Quarterly average return: Portfolio 2 = 15.87%, S&P500 = -8.49%

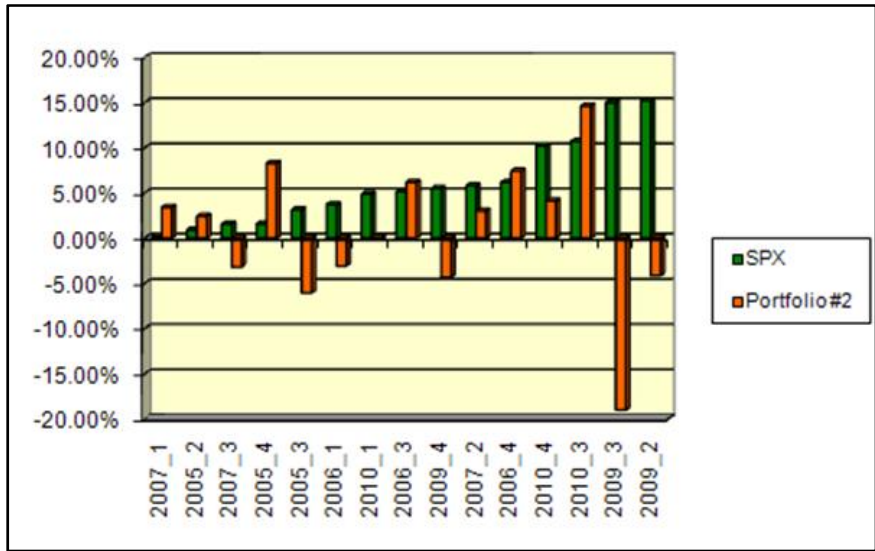


Fig.8: All positive quarterly returns of S&P500 and AOA Portfolio2.
 Quarterly average return: Portfolio 2 = 0.63%, S&P500 = 5.98%

To better understand the benefits of Portfolio 2, it might be useful to think of it as a car insurance that pays on average, twice the cost of a car in case of an accident. In addition, this insurance is free, or more precisely, on average, this insurance gives a small positive payoff. AORDA Portfolio 2 acts as an insurance against S&P500 drawdowns that during the past 6 years, as is shown in Fig.7, has grown on average about two times more than the market losses during quarters with negative market returns. And when the S&P500 was rising, has lost no money on average (Fig.8 shows that the average quarterly return on Portfolio 2 is slightly positive in quarters with positive S&P500 returns). AORDA Portfolio 2 is a statistical hedge. When the market goes up, the investor's total wealth goes up as AORDA Portfolio 2 losses are less than market gains on average. When the market goes down, the investor's wealth goes up because AORDA Portfolio 2 on average pays more than the market losses.

The idea behind AORDA Portfolio 3 comes from the fact that a good portfolio, having positive returns in both up and down markets, can be constructed by mixing the S&P500 with AORDA Portfolio 2. The following Table 4 shows that the strategy of investing 50% in the S&P500 and 50% in Portfolio 2, over the recent 6 years, returned an average of 14%, with the largest drawdown 13.52%, and a Sharpe ratio of 1.34.

PORTFOLIO PERFORMANCE CATEGORY	100% in S&P500 and 0% in Portfolio 2	75% in S&P500 and 25% in Portfolio 2	50% in S&P500 and 50% in Portfolio 2
Annualized ROR (%)	0.98	7.63	14.00
Annualized Std. Deviation (%)	16.46	11.36	10.24
Sortino Ratio	0.19	0.99	3.08
Sharpe Ratio	0.14	0.71	1.34
Largest Drawdown (%)	52.56	29.73	13.52

Tab. 4: Mixing Portfolio 2 with S&P500 (period Jan. 2005 - Jan. 2011).

AORDA Portfolio 3 is slightly leveraged compared to the 50%-50% portfolio described in the last column of the Table 4. Portfolio 3 has the following composition: 66% in S&P500 and 34% in Portfolio 2. Leveraging is achieved by using mini-futures or leveraged positions in ETFs.

Fig. 9 shows the value of \$100 invested in January 2005 through January 2011 in AORDA portfolios vs. S&P500. Portfolio 2 “mirrors” the performance of the S&P500, and it is negatively correlated with the S&P500. On the other hand, Portfolio 2 has a quite high positive return (doubling the value every 3 years). Portfolio 2 has properties of long volatility strategy: it achieves high positive return (exceeding market loss) in bear markets and still attains a positive return (on average) in bull markets. Portfolio 3, which is a mixture of the S&P500 and Portfolio 2, performs quite well both in up and down markets. Let us compare the performance of the Short PutWrite Index (see, Fig. 2) and the inverse fund SH (see, Fig. 4) with Portfolio 2 (see, Fig. 9). These three instruments, Short PutWrite Index, SH, and Portfolio 2 are negatively correlated with the market and provide protection against market losses. However, the Short PutWrite Index and SH have a negative return in the long term, but Portfolio 2 has a strong positive return (Portfolio 2 doubles every 3 years).



Fig.9: AORDA Portfolios vs. S&P500.

Table 5, 6, and 7 describe the composition of AORDA Portfolios.

AORDA® Portfolio 1

Goal	Hedging market losses
Correlation with S&P500	Negative
Most favorable scenario	Bear Market
Composition	Up to 50% of available capital in index S&P500 and up to 50% of available capital in Nasdaq 100, remaining capital is invested in cash
Leverage	None

Tab. 5: Description of AORDA Portfolio 1

AORDA® Portfolio 2

Goal	Hedging market losses
Correlation with S&P500	Negative
Most favorable scenario	Bear Market
Composition	Up to 50% with leverage 2 in S&P500 and up to 50% with leverage 2 in Nasdaq 100, remaining capital is invested in cash
Leverage	Maximum leverage is 2

Tab. 6: Description of AORDA Portfolio 2

AORDA® Portfolio 3

Goal	Growth
Correlation with S&P500	Positive
Most favorable scenario	Bull Market
Composition	Up to 33.5% of available capital with leverage 2 in S&P500; up to 33.5% of the available capital with leverage 2 in Nasdaq 100; 33% of available capital with leverage 2 is always fully invested in S&P500 index
Leverage	Maximum leverage is 2

Tab. 7: Description of AORDA Portfolio 3

The following Table 8 presents actual trading track record of AORDA Portfolios.

	Portfolio #1	Portfolio #2	Portfolio #3	S&P 500 Index
Since Inception	104.92	299.63	162.03	9.51
Inception/Annual	12.34	25.19	16.91	1.48
2011 (Jan-Feb)	1.70	3.40	5.40	5.53
2010	19.61	42.15	31.22	12.78
2009	-8.08	-15.96	3.40	23.45
2008	39.63	90.56	15.15	-38.49
2007	5.89	11.46	8.03	3.53
2006	16.06	34.15	32.00	13.62
2005	6.81	13.54	11.60	3.00

Tab. 8: AORDA Portfolios realized returns (January 2005 – February 2011).

Table 9 gives performance characteristics of AORDA Portfolios.

PERFORMANCE CATEGORY	Portfolio #1	Portfolio #2	Portfolio #3	S&P 500	NDX	DJI
Cumulative Return (%)	104.92	299.63	162.03	9.51	45.02	13.39
Annual Compounded Rate of Return (%)	12.34	25.19	16.91	1.48	6.21	2.06
Sharpe Ratio (Risk Free = 0%)	1.23	1.22	1.22	0.17	0.40	0.21
Sortino Ratio (Risk Free = 0%)	2.85	2.88	2.31	0.23	0.57	0.28
Correlation with S&P500 Index (%)	-40.37	-39.36	19.59	100	90.55	97.52
Maximum Portfolio Drawdown (%)	15.06	28.16	14.75	52.56	50.11	49.30
Annual Standard Deviation (%)	9.87	20.13	13.66	16.39	20.63	15.23
Annual α -coefficient (%)	12.84	25.92	16.18	n/a	4.99	0.67

Tab. 9: Comparative performance of AORDA Portfolios (January 2005 – February 2011). Negative/Low correlation with S&P500.

AORDA Portfolios are composed of S&P500 and Nasdaq100 (NDX) indices. The following Section 6 explains how weights for S&P500 and Nasdaq100 are obtained. Given these weights, Portfolio 2 makes use of leverage 2 to invest up to 100% of the capital in the S&P500 and up to 100% of the capital in the Nasdaq100; the remaining capital, if any, is in cash. Portfolio 1 is a unleveraged version of Portfolio 2, which invests up to 50% of the capital in the S&P500 and up to 50% in the Nasdaq100. Portfolio 3 places 66% of capital in the S&P500 and 66% of capital in Portfolio 2.

Suppose the weights are: 25% to S&P500 and 10% to Nasdaq100. Then, if the initial capital is 100, Portfolio 1 equals: 25% in ES (or SPY), 10% in NQ (or QQQQ), and 65% in cash; Portfolio 2 equals: $2*25=50\%$ in ES (or SPY), $2*10=20\%$ in NQ (or QQQQ), and $(100-50-20)\% = 30\%$ in cash; Portfolio 3 equals: $1.32*25\%=33\%$ in ES (or SPY) plus 66% constant position in ES (or SPY) totaling 99% in ES (or SPY); $1.32*10\%=13.2\%$ in NQ (or QQQQ), and $(100-99-13.2)\% = -12.2\%$ in cash. Portfolio 3 keeps 66% constant exposure to S&P500 and combines it with 66% position in Portfolio 2.

AORDA Portfolios invest in highly liquid mini-futures or ETFs for which prices are readily available. AORDA Portfolios are marked-to-market daily and portfolio returns are precisely calculated. Returns are not “artificially smoothed” as is typical for strategies that invest in illiquid or over-the-counter products. Smoothing returns implies the use of subjective processes of evaluation instead of mark-to-marketing a portfolio. That is a common practice for many hedge funds, and usually results in differences between portfolio evaluation and what the market is willing to pay for it when the fund needs to liquidate its assets.

Portfolio 3 is an equally weighted mix of Portfolio 2 and S&P500. If the strategy is implemented with mini-futures contracts, it is possible to leverage up this strategy and get a higher expected return and higher risk. High return seeking investors at American Optimal Advisors invest to 1.8*(AORDA portfolio 3). The Portfolio 3 is implemented at Interactive Brokers (IB) with completely automatic day trading procedure making two trades per day (opening position in mini-futures ES and NQ in the middle of a trading day and closing positions at the end of the trading day). Moreover, the long constant position in S&P500 is replaced by CBOE PutWrite Index which has a lower risk than the S&P500 (losses of PutWrite Index are always smaller than the losses of S&P500).

The 1.8*(AORDA portfolio 3) at IB is implemented as follows:

$$1.8*(\text{AORDA portfolio 3}) = 1.2*(\text{AORDA Portfolio 2}) + 1.2*(\text{PutWrite Index}) .$$

The average return of 1.8*(AORDA portfolio 3) over 6 years (gross of fees) equals about 30% and the maximum drawdown equals about 30%. The PutWrite Index is a quite profitable strategy taking advantage of the overpriced downside protection with put options. This strategy is hedged with Portfolio 2 positive returns when the market goes down and the PutWrite Index has negative returns.

The performance of AORDA Portfolio 2 has been independently verified by Thetaresearch.com since October 2005 and AORDA Portfolio 3 since January 2009. Thetaresearch.com recorded the performance of two self-financed accounts created to enable tracking of Portfolios 2 and 3 managed by AOA. Thetaresearch.com reports performance of these two accounts to the database (net of about 3% management fee per year). It is possible to get a free login to Thetaresearch.com to view the historical performance of AORDA portfolios by placing e-mail address to this link: <http://manager.thetaresearch.com/guestsignup.mgr?mgrid=381>

6. CVaR Approach to Portfolio Selection

AORDA Portfolios are based on formal optimization methods and the risk management technology designed to constrain risk with the objective of growing the value of assets. The solution of an optimization problem assigns weights to S&P500 and NDX indices and to cash. The investment methodology involves two main steps:

- 1) building forward looking scenarios for the portfolio instruments;
- 2) applying the Conditional Value-at-Risk (CVaR) risk management technology to convert scenarios to optimal weights.

Worldwide financial and macroeconomic data are used to build forward-looking scenarios. Current and historical pricing information are used to identify changes in the domestic and foreign equity and fixed income markets, and in the underlying assets themselves. This involves the use of various technical indicators, such as moving averages and trend-lines, among others. Various fundamental data such as economic conditions, industry outlook, interest rates, and the political environment are also considered.

Risk control is achieved by using state-of-the-art techniques of portfolio optimization under uncertainty. Nowadays, approaches to estimating tail risk have gained popularity. Portfolios are selected on the basis of percentile characteristics such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR). Extensive research has been done on the properties of these risk functions as risk management tools and on their use in portfolio optimization. AORDA Portfolios are based on techniques described by Rockafellar and Uryasev (2002) that pioneered the CVAR risk management optimization technology. In this way, scenario outcomes designed to reflect up-to-date forecast of future states are incorporated in the portfolio weights selection.

AORDA portfolio weights are obtained by solving the following optimization problem: the portfolio expected return is maximized with a constraint on Conditional Value-at-Risk (CVaR). The CVaR risk measure evaluates the average of the worst-case 10% scenario losses (confidence level is equal to 90%). The obtained portfolios are back-tested on historical data in various market conditions. The portfolio optimization problem that maximizes the expected portfolio return under CVaR-risk constraint has the form:

Maximizing expected return

$$\max \text{ExpectedReturn}(\vec{x})$$

Risk constraint (90%-CVaR is bounded)

$$\text{CVaR}_{90\%}(\vec{x}) \leq w$$

Budget constraint

$$\sum_{i=1}^I x_i = U$$

Bounds on exposures

$$l_i \leq x_i \leq u_i$$

where:

I = number of instruments in the portfolio, $i=1, \dots, I$;

\vec{x} = vector of decision variables, i.e., portfolio weights assigned to each instrument;

w = upper bound for CVaR risk;

U = available capital;

l_i = lower bound on exposure to instrument i ;

u_i = upper bound on exposure to instrument i .

The stated CVaR minimization problem is solved with Portfolio Safeguard (PSG), a software package designed for problems with up to 1 million scenarios and mixtures of normal distributions (see, American Optimal Decisions (2009)). The idea of PSG is that nonlinear functions, such as CVaR and others, are pre-coded and included in the library of available functions. Therefore, a code for solving CVaR optimization problems, as well as other financial optimization problems, includes only several lines. PSG can be run in several environments: Microsoft Windows Shell, MATLAB, Text (Run-File), and C++. Many real-life financial optimization problems (with codes, underlying data, and solutions) are posted online for download, see Test Problems (2010). Short term (several hours) return distributions exhibit heavy tails. Therefore, a very large number of scenarios are needed to correctly model return distributions. A popular approach to model heavy tail distributions is to use a mixture of normal distributions. CVaR test optimization problems based on scenarios are posted at

http://www.ise.ufl.edu/uryasev/testproblems/case_studies/CS_Optimization_Beyond_Black_Litterman/CS_Optimization_Beyond_Black_Litterman.htm

and Value-at-Risk (VaR) and CVaR optimization problems based on scenarios and mixtures of normal distributions are posted at

http://www.ise.ufl.edu/uryasev/testproblems/case_studies/CS_Portfolio_Credit-Risk_Optimization/CS_Portfolio_Credit-Risk_Optimization.htm

You can install PSG, download the codes from the mentioned websites, and solve VaR and CVaR optimization problems.

7. Conclusion

The volatility of the market is itself highly volatile, and it is higher during market drawdowns. Several different strategies can be put in place to hedge a portfolio from market movements. Investment strategies that short volatility have good performance during periods of low volatility (in up and side-ways markets), but are exposed to extreme losses should the volatility sharply increase (in a declining market). Examples of short volatility strategies are short put options, and the PutWrite Index. Many hedge fund styles short volatility.

Buying long put options is an effective way to protect a portfolio in declining markets, but put options are expensive and can cause negative returns in the long run. The Short PutWrite index, discussed in section 2, significantly deteriorated the performance of a long position in the underlying S&P500 after a large market drop. Portfolio insurance techniques are similar to a long put strategy. These techniques are expensive and can cause snowball effects and liquidity problems, as demonstrated in the crash of October 1987.

Inverse ETFs provide good protection against market drops on a short term basis. However, these ETFs do not have good hedging characteristics over the long term. The inverse ETFs may completely wipe out the positive returns of long portfolios, which they are supposed to hedge.

AORDA Portfolio 2 is a long volatility strategy that provide a statistical hedge for market drawdowns. This strategy performs especially well during bear markets while delivering (on average) small positive returns in bull markets. AORDA Portfolio 3 is an equally weighted mix of Portfolio 2 and the S&P500 and it performs well both in up and down markets. AORDA Portfolios are based on a completely automatic quantitative trading strategy combining long and short position in highly liquid instruments (futures, ETFs) tracking the S&P500 and Nasdaq 100 indices. AORDA Portfolios are based on CVaR formal risk optimization technology designed to constrain risk with the objective of growing asset values. If the strategy is implemented with mini-futures contracts, it can be leveraged to get a higher expected return but with higher risk. Moreover, in Portfolio, 3 by replacing the constant position in S&P500 with the PutWrite Index, it is possible to decrease the risk and improve returns (loss on the PutWrite Index is always smaller than the loss on the S&P500).

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